Sub-wavelength Coherent Imaging of a Pure-Phase Object with Thermal Light

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We report, for the first time, the observation of sub-wavelength coherent image of a pure phase object with thermal light. We demonstrate that ghost-imaging scheme (GI) recovers *amplitude* transmittance of objects rather than the transmitted *intensities* as the classical Coherence-Function Imaging (CFI) scheme does. Also, the understanding of whether the entanglement is a prerequisite for GI is sharpened:Only entangled two photon state contributes to the joint detection,whereas the entangled light beam source is not a prerequisite.

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In many imaging circumstances, phase information about objects plays a role as well as or even more important than intensity does, for example, when the objects are pure-phased, that is, highly transparent and absorb little light, imaging can not be simply realized by the transmitted or reflected intensity information of thermal lights. Although phase distribution about an object can be retrieved from its Fourier-transform diffraction pattern was firstly proposed by Sayre [1] and dedicated efforts described in the works like [2] demonstrated and developed the techniques, the efforts seems to be in vain if diffraction imaging applications were in hard x-ray, γ-ray, or other wavelengths where no effective lens or/and no coherent source is available. Recent works [3, 4] reported a new version of the landmark Hanbury Brown and Twiss (HBT) experiment [5] and gave lensless Fourier-transform pattern of a Youngs double slit with thermal sources, but the phase information is not yet mentioned because the object they used is amplitude-only. In fact, as we will discuss later, the classical HBT scheme of Coherence-Function Imaging (CFI),in which optical fields at both detect plane have passed through the object, will be invalid for retrieving phase knowledge about an object. Since middle years of last decades, ghost imaging (GI) has been enthusiastically studied[6, 7, 8, 9, 10, 11, 12, 13, 14], here, the reason for the term *ghost* used is that the image of an object, diffractive or geometrical, would appear as a function of the position at one path that actually never pass the object, and this unique feature is key difference from classical CFI. Whether the entangled beams was a prerequisite once have been hotly debated. It is generally accepted now that classical thermally emitted light can be used for GI and quantum entangled beams is not a prerequisite. In fact, as we can see as follows, the general acceptation dose not necessarily mean that entanglement plays no role in GI.

The physics behind the joint detection in plane x_1 and plane x_2 , as Fig.1 shows, can be explained in simplicity as follows: The two-photon amplitude described by state vector $|A\rangle$ reduces to weighted sum of the following three normalized basis states: $|\alpha\rangle = |vac, j\rangle$, two-photons both

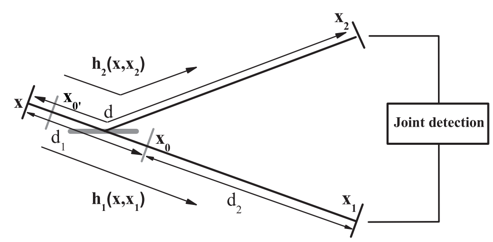

FIG. 1: Setup schemas for experiment.$h_k(x, x_k)$ refers to the impulse response functions of both arms k from source plane x to detection plane x_k;$k = 1, 2$.

reflected by BS; $|\beta\rangle = |m, n\rangle$, one photon reflected by the BS and the other transmitted the BS; and $|\gamma\rangle = |i, vac\rangle$, two-photons both transmitted the BS. $i.e.|A\rangle = \frac{1}{2}|\alpha\rangle + \frac{1}{\sqrt{2}}e^{i\theta_1}|\beta\rangle + \frac{1}{2}e^{i\theta_2}|\gamma\rangle$. In the equation, $\theta_{1(2)}$ is the phase of complex weight for state $|\beta\rangle(|\gamma\rangle)$ relative to state $|\alpha\rangle$.The expression of $|\alpha\rangle$ and $|\gamma\rangle$ can be expressed as:

$$|\alpha\rangle = |vac\rangle_1|j\rangle_2, \tag{1}$$

and

$$|\gamma\rangle = |i\rangle_1|vac\rangle_2, \tag{2}$$

but as for the characters of identical bosons, the two-photon state of $|\beta\rangle$ must be expanded in this way:

$$|\beta\rangle = |m\rangle_1|n\rangle_2 + |n\rangle_1|m\rangle_2. \tag{3}$$

The subscripts 1 and 2 of state vector $|\rangle$ refer to detecting plane x_1 and x_2, respectively. Obviously, the expression of state $|\beta\rangle$ state the entanglement between the transmitted and reflected but undistinguished photons.

If we define $E^{(\pm)}_{1,2}(t_{1,2}r_{1,2})$as the positive-frequency and negative-frequency components of the field at time-spatial point t_1x_1 and t_2x_2, the two-photon coincidence rate obtained by joint detection at the two time-spatial point, interpreted as a probability per unit $(time)^2$ that one photon is recorded at x_1 at time t_1 and another at x_2 at time t_2, say, the square module of two-photon amplitude $\psi(t_1x_1, t_2x_2)$ presented by $|A\rangle$ has been described by second-order Glauber correlation function [15]: $G^2(t_1r_1, t_2r_2) = Tr\{\rho E_1^{(-)}(t_1r_1)E_2^{(-)}(t_2r_2)E_2^{(+)}(t_2r_2)E_1^{(+)}(t_1r_1)\}$, where the density operator is defined as the average outer product of state $|A\rangle$: $\rho = \{|A\rangle\langle A|\}_{av}$, *i.e.* ,

$$\rho = \frac{1}{4}|\alpha\rangle\langle\alpha| + \frac{1}{2}|\beta\rangle\langle\beta| + \frac{1}{4}|\gamma\rangle\langle\gamma|. \quad (4)$$

Subsisting Eq.(1-4)into second-order Glauber correlation function, and note that $E_{1,2}^{(+)}(t_{1,2}r_{1,2})|vac\rangle = \langle vac|E_{1,2}^{(-)}(t_{1,2}r_{1,2}) = 0$, we find that only the entangled state $|\beta\rangle$ contributes to the joint detection:

$$G^{(2)}(t_1r_1, t_2r_2) \propto \langle\beta|E_1^{(-)}(t_1r_1)E_2^{(-)}(t_2r_2) E_2^{(+)}(t_2r_2)E_1^{(+)}(t_1r_1)|\beta\rangle. \quad (5)$$

Glauber's quantum detection theory shows that right side of Eq.(5) is proportional to $\langle I_1(r_1t_1)I_2(r_2t_2)\rangle$[16, 17], where $I_{1,2}(r_{1,2}t_{1,2})$ is the instant intensity for analog measurement at time-spatial points t_1r_1 and t_2r_2. Based on this theory, the following experiment was carried out in the regime of large number of photons to illustrate the behaviors of two-photon interference.

Previous works[12, 13] have stated that if one defines $\Delta I^{(2)}(t_1r_1, t_2r_2) = \langle I_1(r_1)I_2(r_2)\rangle - \langle I_1(r_1)\rangle\langle I_2(r_2)\rangle$, and assumes the time window $\Delta t = t_2 - t_1 = 0$ in the experiment, the impulse response functions for both arm $h_k(x, x_k)$,$k = 1, 2$ would be embedded into:

$$\Delta I^{(2)}(x_1, x_2) \propto |\int_x h_1^*(x', x_1)h_2(x, x_2) < E^*(x')E(x) > dx'dx|^2. \quad (6)$$

The physics behind the above theoretical expression is that the modulation to the two-photon amplitude $\psi(t_1x_1, t_2x_2)$ by the objects may be measured by correlation of intensity fluctuations.

The pure phase object we used is prepared by etching two grooves with width of $150\mu m$ and separating them by a $150\mu m$ un-etched area on a piece of $0.75mm \times 10mm$ square quartz glass (JGS1). The other two un-etched areas with width of $150\mu m$ are left symmetrically. Since we are using Nd: YAG laser with the wavelength of $\lambda = 0.532\mu m$, the depth of two grooves is arranged to be $\lambda/2(n-1) = 0.532/2(1.57-1) = 0.46\mu m$ to form phase differences of $\Delta\phi = \pi$ from those un-etched area. The thermal light source at plane x is simulated by projecting a beam of laser pulse (frequency doubled Nd: YAG impulse laser,$\lambda = 0.532\mu m$) onto a slow-round ground glass disk. Away from thermal light source plane x $40mm$, we place a beam splitter to form two arms for imaging system as Fig.1 shows. The pure phase object is placed on plane x_0 away from source plane x $d_1 = 60mm$. Charge Coupled Device camera CCD-1 is placed at plane x_1 on the same arm after the object $d_2 = 75mm$. Correspondingly, we place CCD-2 at plane x_2 on the other arm, its distance from the thermal light source x is $d = 135mm$. The parameters here are thus arranged to meet with condition $d = d_1 + d_2$, which was required by the theoretical prediction in Ref.[13]. The light sport diameter on ground glass is chosen as $\sigma = 3mm$ in order to ensure linear dimensions of the coherence area[18] of the pseudo-thermal light field across the object plane $D \approx \lambda d_1/\sigma = 0.532\times 60/3 = 10.64\mu m$ to be much smaller than the pure object's feature size of $150\mu m$. The exposure time of two CCD cameras is set to $1ms$. Totally about 10,000 frames of independent two-dimensional instant intensity distribution data of speckle fields in the planes of x_1 and x_2 , say, ensembles of $I_1(x_1)$ and $I_2(x_2)$, are recorded to prepare for correlation operation. The laser pulse shooting, data acquisition and their recording process are synchronized and accomplished by computer.

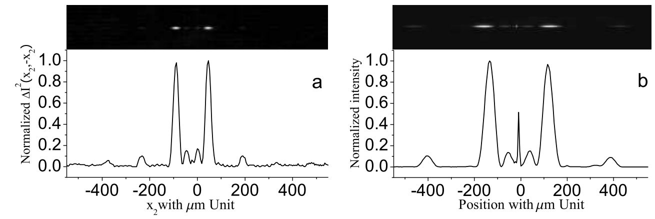

FIG. 2: **a**. Gray scale conversion and its profile of $\Delta I_2(x_2, -x_2)$; **b**. the Fraunhoufer diffraction pattern of the pure phase object obtained by coherent $2f$ system with $f = 75mm$ and $\lambda = 0.532\mu m$.

Choosing symmetric positions x_2 and $x_1 = -x_2$ to calculate $\Delta I^{(2)}(x_1, x_2)$ with preserved data $I_1(x_1)$ and $I_2(x_2)$, we find what we obtain (Fig.2.a) shares the same pattern in Fig.2.b, but have double coordinate scales just as

$$\Delta I^2(x_2, -x_2) = |\ \mathcal{F}\{\frac{2\pi[x_2 - (-x_2)]}{\lambda d_2}\}\ |^2 = |\ \mathcal{F}\{\frac{2\pi x_2}{\frac{\lambda}{2}d_2}\}|^2 \quad (7)$$

anticipated in our previews theoretical work[13]. The result(Fig.2.a) shows a sub-wavelength interference pattern equals to the Fraunhoufer diffraction pattern(Fig.2.b) of the same pure-phase object produced by $2f$ system(with $\lambda = 0.532\mu m$ and $f = 75mm$), but with half of the wavelength of coherent illumination.

The experimental setup is compatible with the classical HBT schemed CFI system if we move the object from plane x_0 to $x_{0'}$(Fig.1). After that we repeat the experiment, the result turns out to be a diffraction pattern of the limited aperture of the object (Fig.3), and contains no any information of the pure phase object as can be inferred from Eq.(6) that if the same complex amplitude transmittance function of object contained in both impulse response function h_1 and h_2, its phase knowledge will be lost and remained only module information about it because of the factor of $h_1^*(x', x_1)h_2(x, x_2)$.

Thus we find the other essential difference from re-

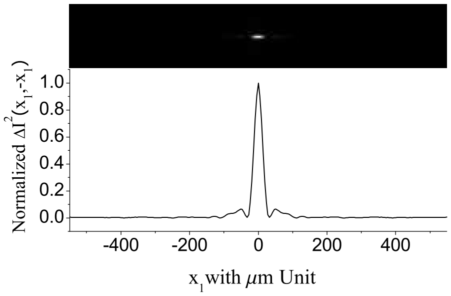

FIG. 3: HBT schemed setup recovers a diffraction pattern of the limited aperture of the object and totally not of the pure object itself's at all.

cently reports about Sub-wavelength interference with thermal light [3, 4] is what GI type of imaging system in our experiment setup retrieved was the complex amplitude transmittance knowledge of the object rather than the transmitted intensity as the HBT schemed CFI does.

By providing the experimental evidence for sub-wavelength diffraction pattern of pure phase object illuminated by the thermal light without entanglement, not only have we retrieved the amplitude transmittance knowledge about pure-phase object, but more basically, sharpened the understanding the role of entanglement in GI: Although the entangled light beam source is not a prerequisite, only entangled two photon state generated by the beam splitter contributes to the joint detection, as Eq.(5)demonstrated.

retrieved patterns as the module distribution of two-photon amplitude $\psi(t_1r_1, t_2r_2)$ at the selected time-spatial point. Apart from the unique *ghost* feature of the experiment result, attention shall also be paid again to the similarity and the difference between GI and the classical HBT type of CFI: They both lenslessly retrieve diffraction patterns by correlation function whereas GI recovers knowledge of complex transmittance about objects rather than the transmitted intensities as CI does.

Obtains of sub-wavelength interference pattern suggest that diffraction limit can be broken through by using two-photon absorption (TPA) media if we find ways to fold the symmetric planes of x_1 and x_2 into the same one. This feature appeals to quantum lithography which was widely discussed[19].

Unlike the other ghost imaging and ghost diffraction experiments, a pulsed thermal-like source is used instead of a continuous one, the intensity correlation can be measured even the exposure time of CCD camera is much longer than pulse width of the source. This unique impulse feature of our thermal light source enlightens us on the issue for recording intensity fluctuation by slow detector. This way suggests achieving to record intensity fluctuating much faster than the respond speed of the detecting system. As for hard x-ray imaging, there is a potential applicability to record the femto-second fluctuations of intensity by using of detecting system with response speed in nanoseconds.

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[1] D. Sayre, Imaging Processes and Coherence in Physics, Springer Lecture Notes in Physics Vol. 112 (Springer-Verlag, Berlin, 1980), p. 229.

[2] J. R. Fienup,Appl.Opt.**21**,2758(1982); J.Cheng, S. Han, J. Opt. Soc. Am. A **18**, 1460-1464(2001); V.Elser, J. Opt. Soc. Am. A. 20, 40-55 (2003).

[3] J. Xiong *et al.*, Phys. Rev. Lett., **94**, 173601 (2005).

[4] G. Scarcelli, A. Valencia and Y. Shih, Europhys. Lett., **68**, 618(2004).

[5] R. Hanbury Brown and R. Q. Twiss, Nature (London) 177, 27 (1956).

[6] A. V. Belinsky and D. N. Klyshko, Sov. Phys. JETP **78**, 259 (1994).

[7] A. F. Abouraddy, B. E. A. Saleh, A. V. Sergienko, and M. C. Teich, Phys. Rev. Lett. **87**, 123602 (2001).

[8] R. S. Bennink, S. J. Bentley, and R. W. Boyd, Phys. Rev. Lett. **89**, 113601 (2002).

[9] A. Gatti, E. Brambilla, and L. A. Lugiato, Phys. Rev. Lett. **90**.133603 (2003).

[10] R. S. Bennink, S. J. Bentley, R. W. Boyd, and J. C.Howell, Phys. Rev. Lett. **92**, 033601 (2004).

[11] M. D'Angelo, Y.-H. Kim, S. P. Kulik, and Y. Shih, Phys. Rev. Lett. **92**, 233601 (2004).

[12] A. Gatti, E. Brambilla, M. Bache, and L. A. Lugiato, Phys. Rev. Lett. **93**, 093602 (2004).

[13] J. Cheng and S. Han, Phys. Rev. Lett. **92**, 093903 (2004).

[14] G. Scarcelli, V. Berardi, and Y. Shih, Phys. Rev. Lett. **96**, 063602 (2006).

[15] R. J. Glauber, Phys. Rev., 130, 2529 (1963).

[16] L.Mandel and E.Wolf, *Optical Coherence and Quantum Optics*, (Cambridge University Press, New York, 1995), p.578.

[17] D.F.Walls and G.J.Milburn, *Quantum Optics*(Springer-Verlag,Berlin,1994),p.39.

[18] J.W.Goodman, in *Laser speckle and related phenomena*, Laser speckle and related phenomena,edited by J. C.Dainty (Springer-Verlag, New York, 1984).

[19] S.L.Braunstein *et al.*, in Quantum Electronics and Laser Science Conference, 2001. QELS '01. Techni- cal Digest. (2001), p. 68; M.DAngelo, M. V. Chekhova, and Y. Shih,Phys. Rev. Lett.,**87**,013602 (2001).